Pairs and triplets of entangled microwave photons

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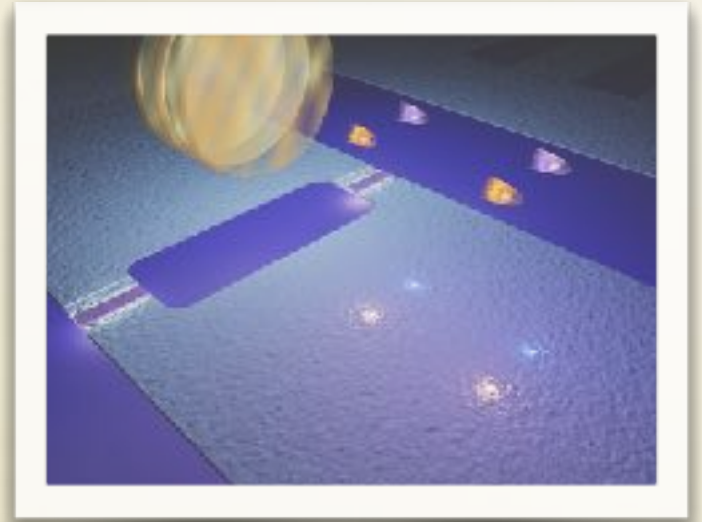

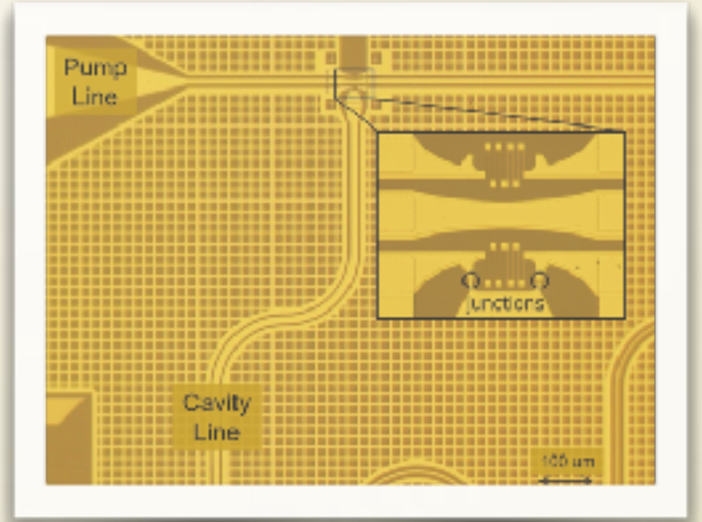

"la Caixa" Foundation

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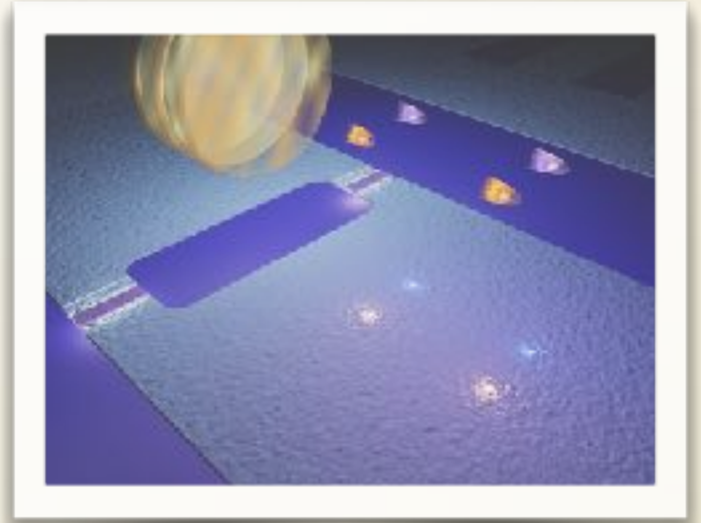
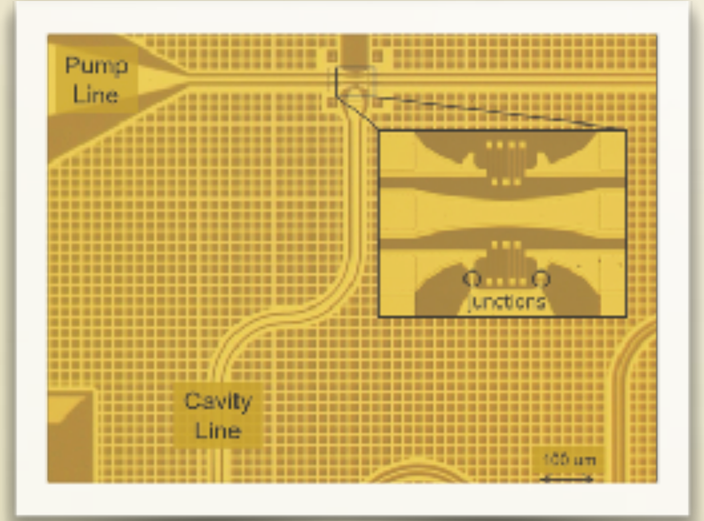
Residencia de Estudiantes, 8/11/2019

In collaboration with…

Chalmers University of Technology
Göran Johansson group (Theory)
Per Delsing group (Experiment)



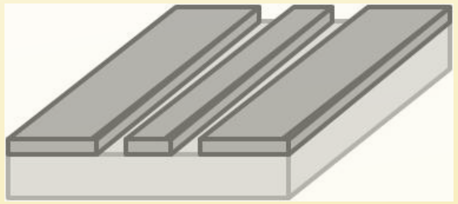


University of Waterloo
Chris Wilson group (Experiment)

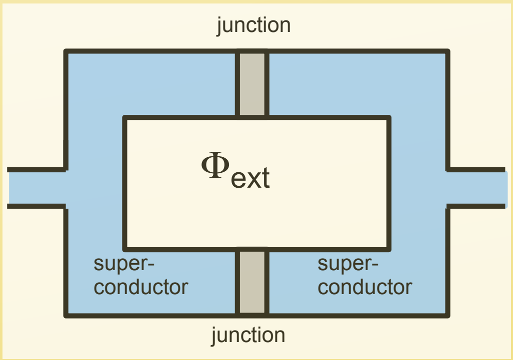
Dynamical Casimir Effect

Particles out of the vacuum!

DCE physical realisation : superconducting circuits



Coplanar waveguide - a transmission line for 1D microwave photons



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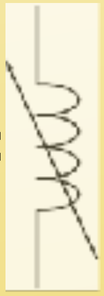
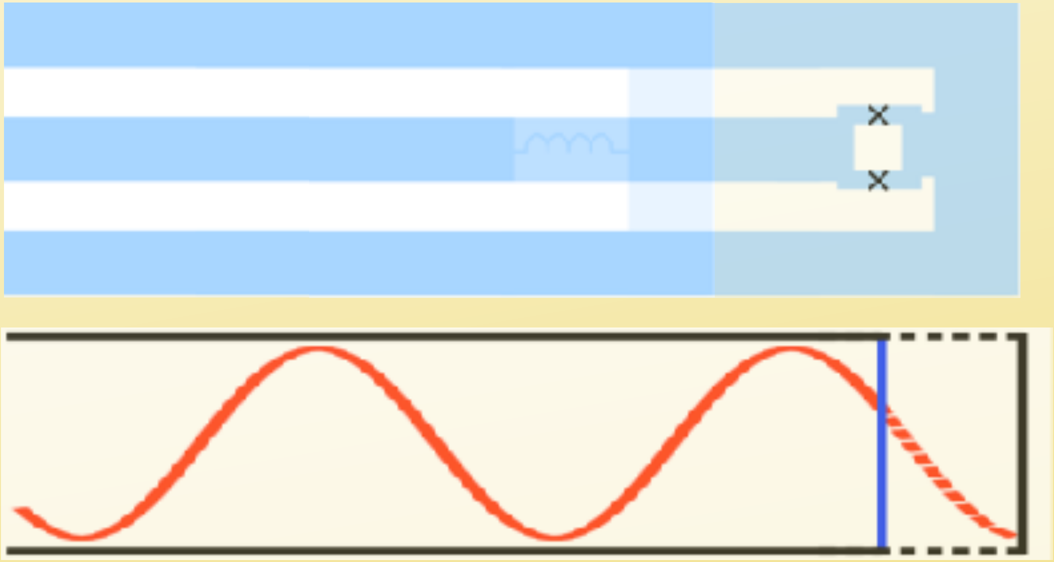
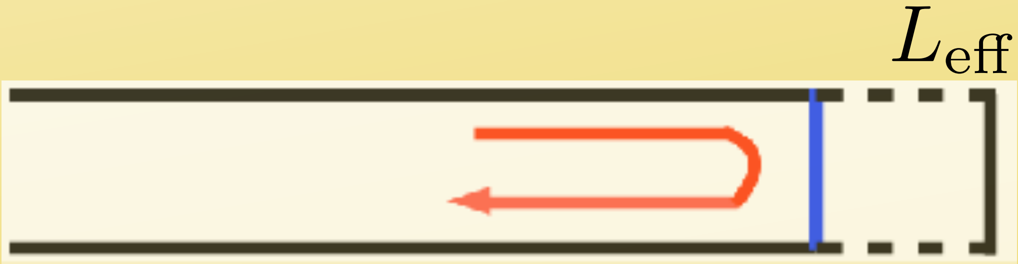


SQUID - superconducting loop interrupted by two Josephson junctions. Can be used as a tunable inductor.

Line with a tunable extra length





$$L_{\mathrm{eff}} = \frac{\text{Termination inductance}}{\text{Inductance per unit length of the line}}$$

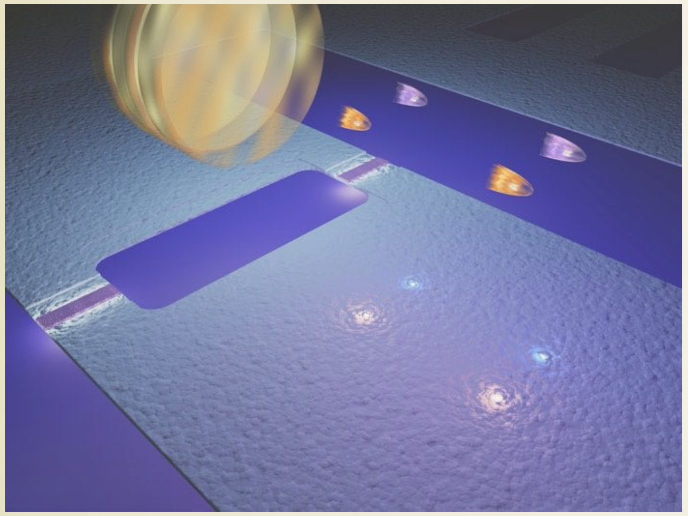
$$\text{SQUID} = \text{Tunable inductance}$$

Dynamical Casimir Effect

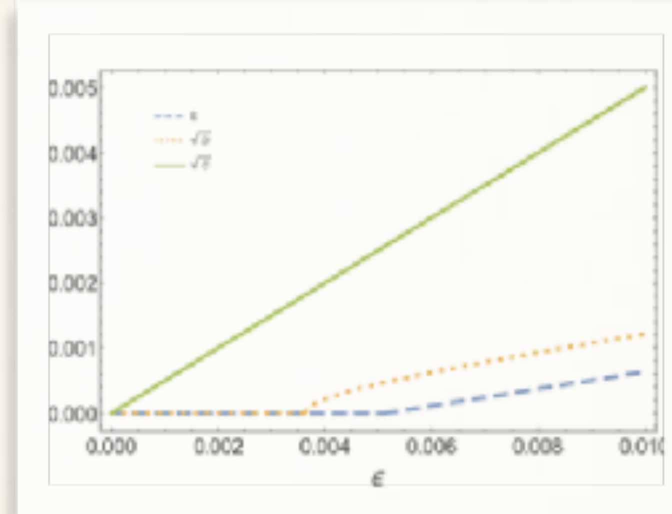
C.M. Wilson, G. Johansson, A. Pourkabirian, M. Simoen, J. R. Johansson, T. Duty, F. Nori and P. Delsing, Nature (2011)

Dynamical Casimir Effect

Particles out of the vacuum!



Pairs of entangled Particles



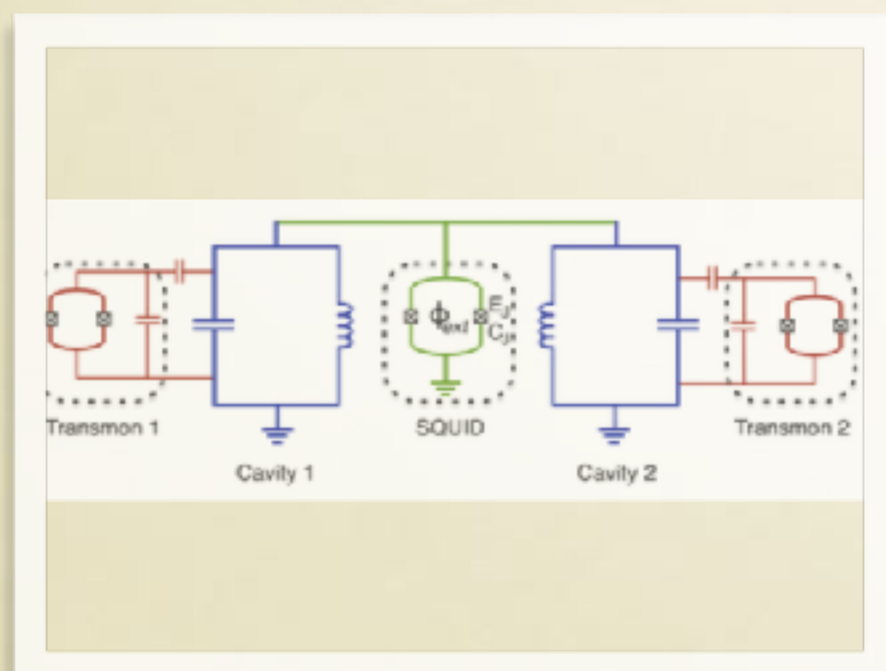
Quantum correlations vs oscillation amplitude

D. N. Samos-Saénz de Buruaga & C.S. Phys. Rev. A 95, 022307 (2017)

C. S, I. Fuentes, G. Johansson, Phys. Rev A 92, 012314 (2015).

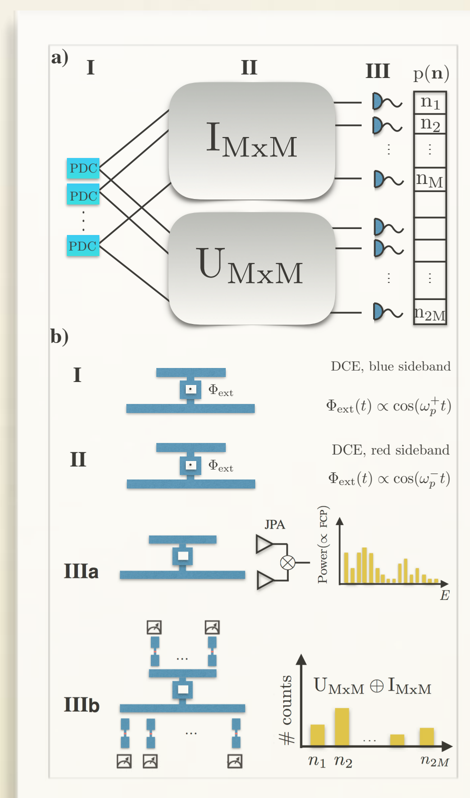
C.S. ,G.Adesso Phys. Rev. A 92, 042107(2015)

DCE entangles Artificial Atoms



Simone Felicetti et al. PRL 113, 093602 (2014)

Boson Sampling



B. Peropadre, C. S, J. Huh Sci. Rep. **8,** 3751 (2018)

Multimode parametric amplification of the vacuum

- Multimode quantum correlations!

- Bipartite and tripartite entanglement out of the vacuum via DCE!

$$\phi = \sum_n \phi_n\, a_n + \phi_n^*\, a_n^\dagger \quad \longrightarrow \quad \hat{\phi} = \sum_n \hat{\phi}_n\, b_n + \hat{\phi}_n^*\, b_n^\dagger$$

Creation and annihilation operators $\longrightarrow$

$$b_n = \sum_m \alpha_{mn}^*\, a_n + \beta_{mn}^* a_n^\dagger$$

mode-mixing

particle creation

- Effective hamiltonian cavity with time-dependent length

$$H_{\text{eff}} = \sum_n \omega_n(t)\left(a_n^\dagger a_n + \frac{1}{2}\right) + \frac{\dot{L}(t)}{L(t)} \sum_n \sum_{j \neq n} \times (-1)^{n+j} \frac{jn}{j^2 - n^2} \sqrt{\frac{n}{j}} (a_n^\dagger a_j^\dagger + a_n^\dagger a_j - a_n a_j^\dagger - a_n a_j),$$

two-mode squeezing

beam splitter

Multimode parametric amplification of the vacuum

$$H_{\rm eff} = \sum_n \omega_n(t)\left(a_n^\dagger a_n + \frac{1}{2}\right) + \frac{\dot{L}(t)}{L(t)} \sum_n \sum_{j\neq n} \times (-1)^{n+j} \frac{jn}{j^2-n^2}\sqrt{\frac{n}{j}}(a_n^\dagger a_j^\dagger + a_n^\dagger a_j - a_n a_j^\dagger - a_n a_j),$$

two-mode squeezing

beam splitter

$$L(t) = L(1+\delta\sin\omega t)$$

$$\delta \ll 1 \rightarrow \frac{\dot{L}}{L} \simeq \delta\omega\cos\omega t$$

Pump (“mirror") frequency: $\omega = \omega_a + \omega_b$ → two-mode squeezer modes a-b

$\omega = |\omega_a - \omega_b|$ → beam-splitter modes a-b

Multimode parametric amplification of the vacuum

Pump ("mirror") frequency: $\omega = \omega_a + \omega_b$ → two-mode squeezer modes a-b

$\omega = |\omega_a - \omega_b|$ → beam-splitter modes a-b

- Two options:

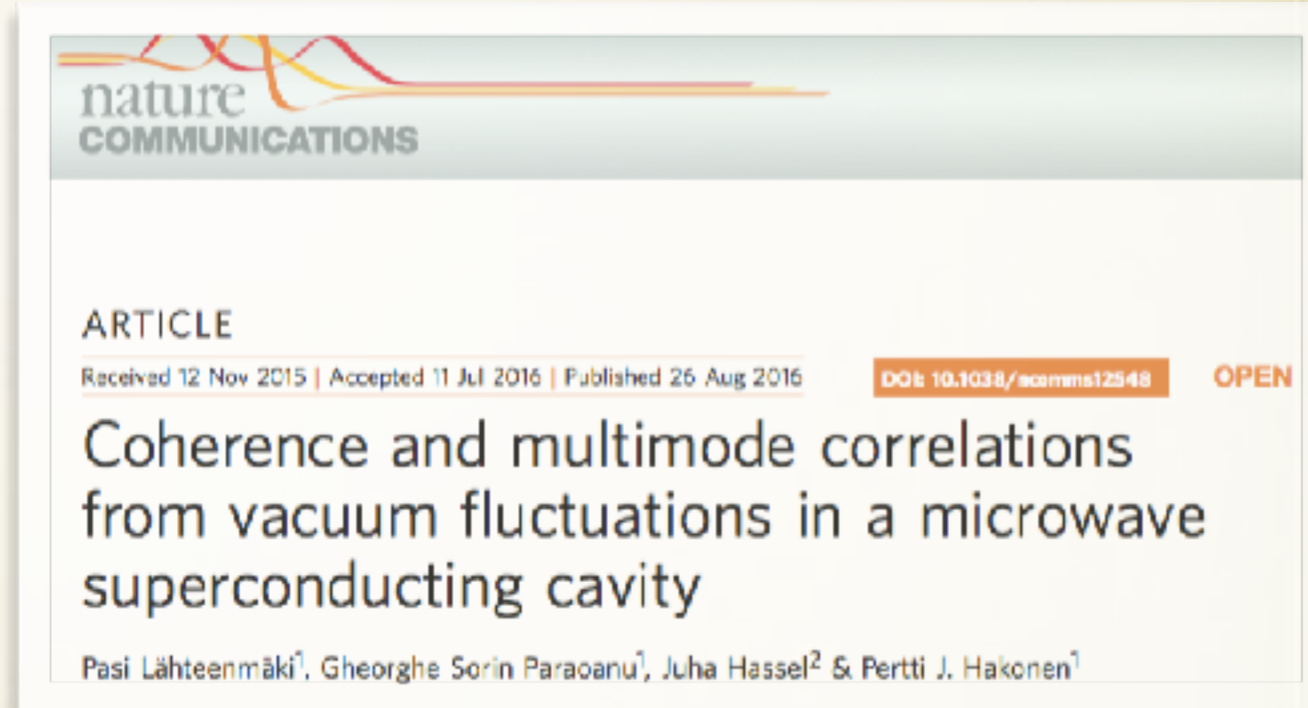
Sequential application of different frequencies to get sequences of two-mode squeezers and beam-splitters.

- Simultaneous application of several frequencies → $e^{A+B} \neq e^A e^B$

→ - Extra interactions

DCE physical realisation : superconducting circuits

- Paraoanu's group in Finland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ARTICLE

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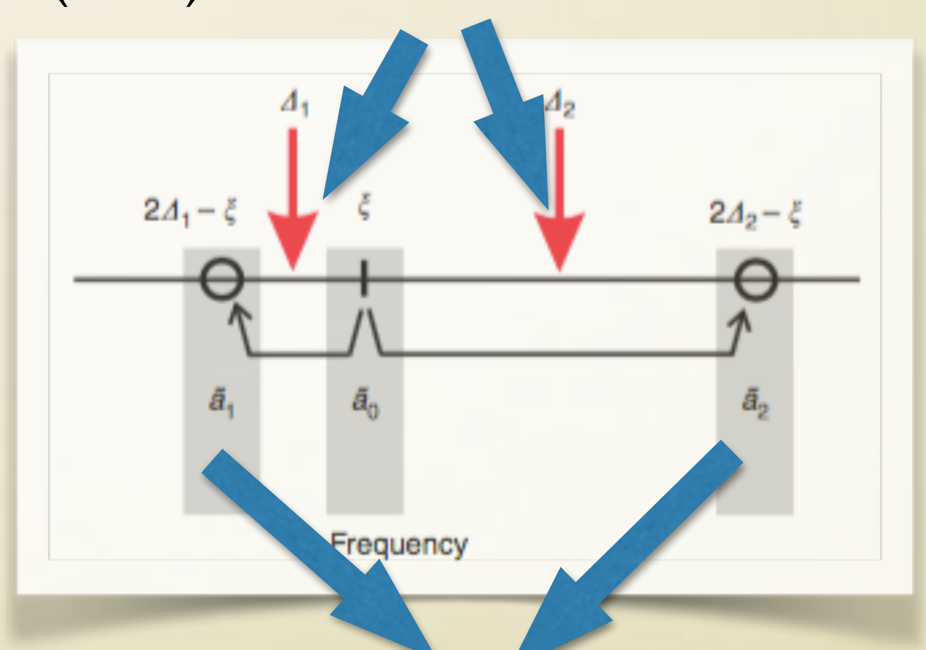
Coherence and multimode correlations from vacuum fluctuations in a microwave superconducting cavity

Pasi Lähteenmäki[1], Gheorghe Sorin Paraoanu[1], Juha Hassel[2] & Pertti J. Hakonen[1]

- Multimode quantum correlations!

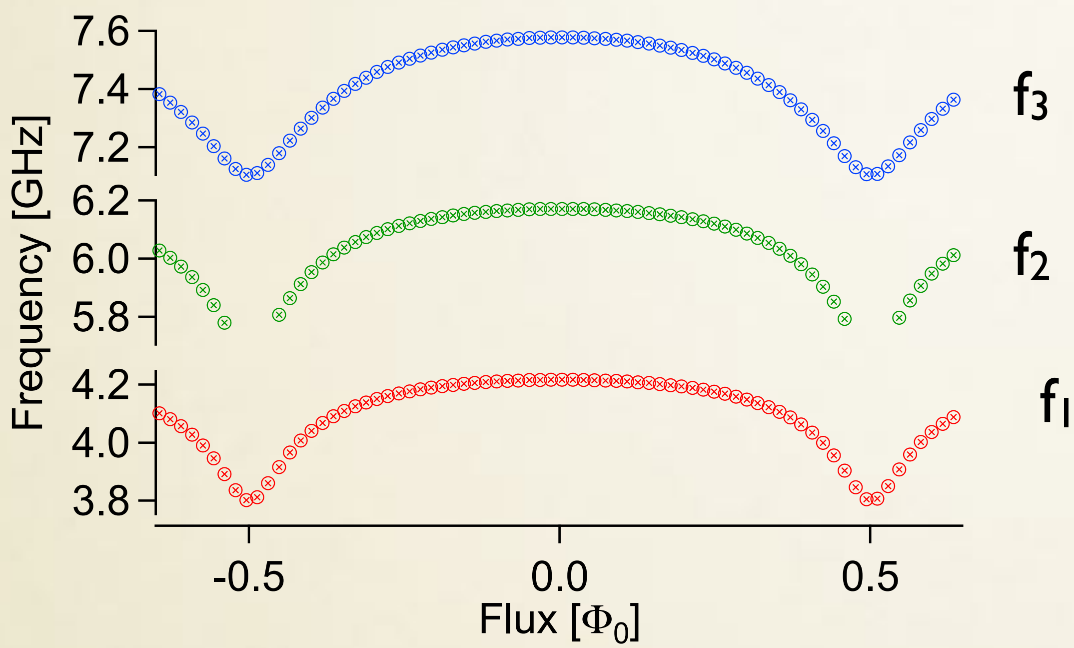
- Theory: D. E. Bruschi, C. S. G. S. Paraoanu, PRA (2017)

TMS



BS

Multimode entangled microwaves



- Bisqueezing scheme:

$$f_{p1} = f_1 + f_2$$

$$f_{p2} = f_2 + f_3$$

- Coupled modes scheme:

$$f_{p1} = f_1 + f_2$$

$$f_{p2} = |f_3 - f_1|$$

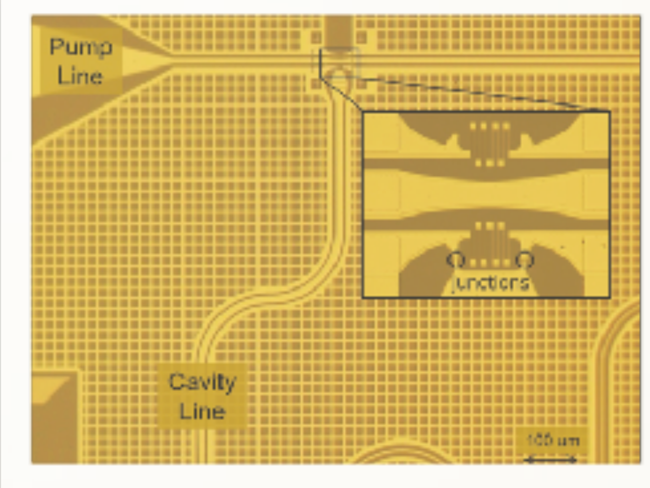
	Frequencies		Entanglement Measures		
Scheme	Modes	Pumps	$\tilde{\nu}_{\min}$	$\mathcal{N}^{tri}$	S
CM	4.20, 6.16, 7.55	10.36, 3.35	0.48 ± 0.002, 0.39 ± 0.002, 0.57 ± 0.002	0.73 ± 0.005	1.49 ± 0.01
BS	4.20, 6.16, 7.55	10.36, 11.75	0.31 ± 0.003, 0.48 ± 0.004, 0.39 ± 0.004	0.94 ± 0.012	1.19 ± 0.01

- Multimode quantum entanglement!

C. W Sandbo Chang, M. Simoen, J. Aumentado, C. S. et al. Phys. Rev. Appl. 10, 044019 (2018).

Third order processes

- Transmission line terminated by asymmetric SQUID $E_{J,1} \neq E_{J,2}$



Interaction Hamiltonian

- Asymmetry in the SQUID
  - $E_{SQ} = E_J(\Phi_{ext}) \cos\left(2\pi \frac{\Phi_{\mathrm{cav}}}{\Phi_0} - \alpha(\Phi_{ext})\right)$
  - asymmetry gives a flux dependent offset $\alpha(\Phi_{ext})$ to Φ_{cav}
  - this gives us access to the cubic term

$$\omega = \omega_a + \omega_b + \omega_c$$

- Third-order SPDC Hamiltonians

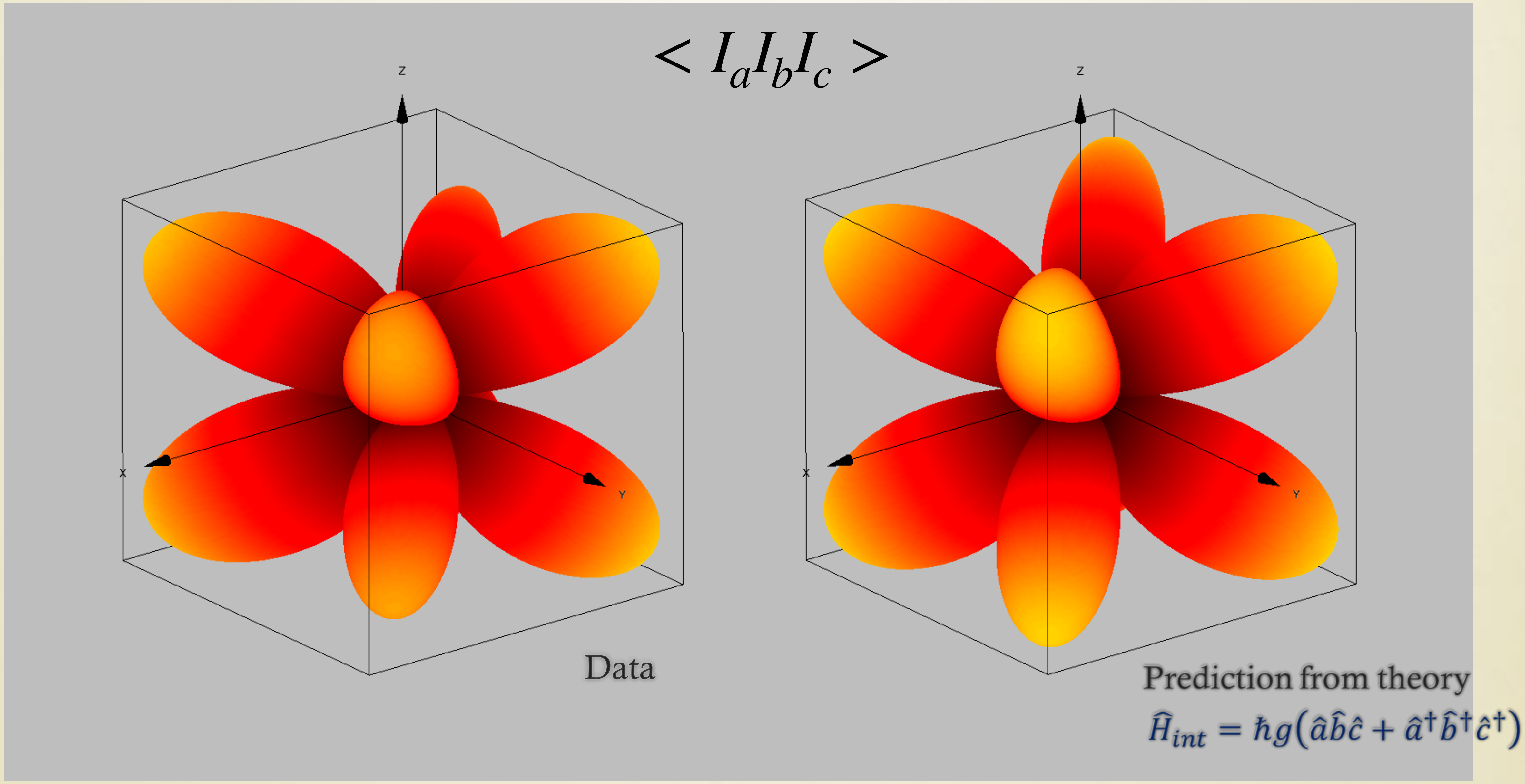
Single Mode: $\hat{H}_{int} = \hbar g\left(\hat{a}^3 + \hat{a}^{\dagger 3}\right)$

Three Mode: $\hat{H}_{int} = \hbar g(\hat{a}\hat{b}\hat{c} + \hat{a}^\dagger \hat{b}^\dagger \hat{c}^\dagger)$

SPDC	Combinations	Frequency [GHz]				Effective Hamiltonians
		Pump	Mode 1	Mode 2	Mode 3	
Single-mode	$f_{p1} = 3 \times f_1$	12.6	4.2	-	-	$\hat{H}_{1M} = \hbar g\left(\hat{a}_1^3 + \hat{a}_1^{\dagger 3}\right)$
Two-mode	$f_{p2} = 2 \times f_1 + f_2$	14.5	4.2	6.1	-	$\hat{H}_{2M} = \hbar g\left(\hat{a}_1^2 \hat{a}_2 + \hat{a}_1^{\dagger 2} \hat{a}_2^\dagger\right)$
Three-mode	$f_{p3} = f_1 + f_2 + f_3$	17.8	4.2	6.1	7.5	$\hat{H}_{3M} = \hbar g\left(\hat{a}_1 \hat{a}_2 \hat{a}_3 + \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_3^\dagger\right)$

C.W. Sandbo Chang, F. Quijandria, C. S, G. Johansson, C. Wilson et al. (to appear in PRX)

Third order processes



- Non-gaussian states → Three-mode correlators →
- Non-gaussian multipartite entanglement?

C.W. Sandbo Chang, F. Quijandria, C. S, G. Johansson, C. Wilson et al. (to appear in PRX)

Non-gaussian states → Three-mode correlators →

Non-gaussian multipartite entanglement?

$$S \equiv \langle(\Delta u^2)\rangle + \langle(\Delta v^2)\rangle \geq 2\min\{|h_i g_i| + |h_j g_j + h_k g_k|\}$$

$$u = h_1 x_1 + h_2 x_2 + h_3 x_3$$

$$v = g_1 p_1 + g_2 p_2 + g_3 p_3$$

Standard criteria are based on two-mode correlators →

Fail to detect entanglement in three-mode SPDC (PRL 120, 043601 (2018))

Need new criteria based on three-mode correlators! →

$$|\langle abc\rangle| > \langle N_i\rangle\langle N_j N_k\rangle$$

→ Full tripartite entanglement

$$\rho \neq \sum_n P_n \rho_n^{(i)} \otimes \rho_n^{(jk)}$$

$$|\langle abc\rangle| \leq \sqrt{\langle N_a\rangle\langle N_b N_c\rangle} + \sqrt{\langle N_b\rangle\langle N_a N_c\rangle} + \sqrt{\langle N_c\rangle\langle N_a N_b\rangle},$$

→ Genuine tripartite entanglement

$$\rho \neq P_1 \rho_1^{(a)} \otimes \rho_1^{(bc)} + P_2 \rho_2^{(b)} \otimes \rho_2^{(ac)} + P_3 \rho_3^{(c)} \otimes \rho_3^{(ab)}$$

A. Agustí, C. S, G. Johansson, C. Wilson et al. (in progress)

$$|\langle abc\rangle| > \langle N_i\rangle\langle N_j N_k\rangle$$

→ Full tripartite entanglement

$$\rho \neq \sum_n P_n \rho_n^{(i)} \otimes \rho_n^{(jk)}$$

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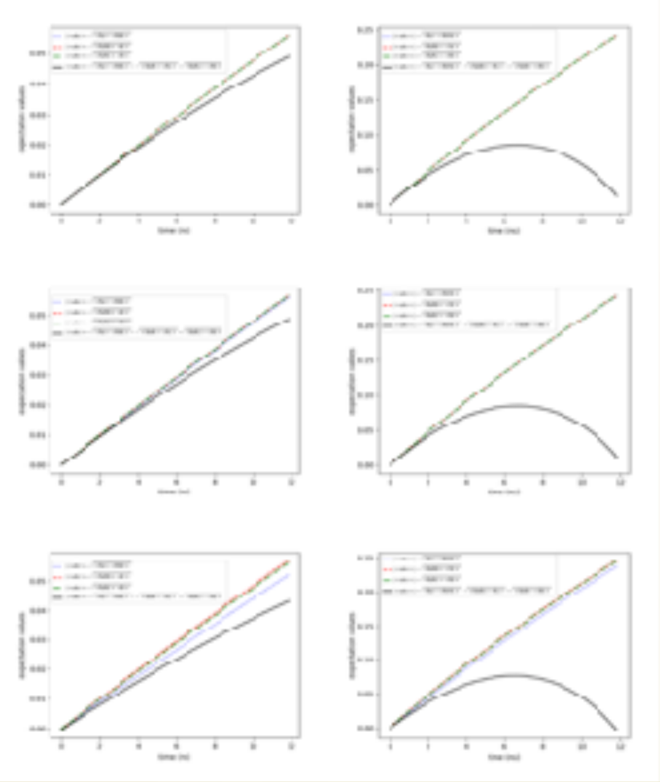
T=10 mk T=30 mk

g = 0.01 GHz

g = 0.05 GHz

g = 0.1 GHz



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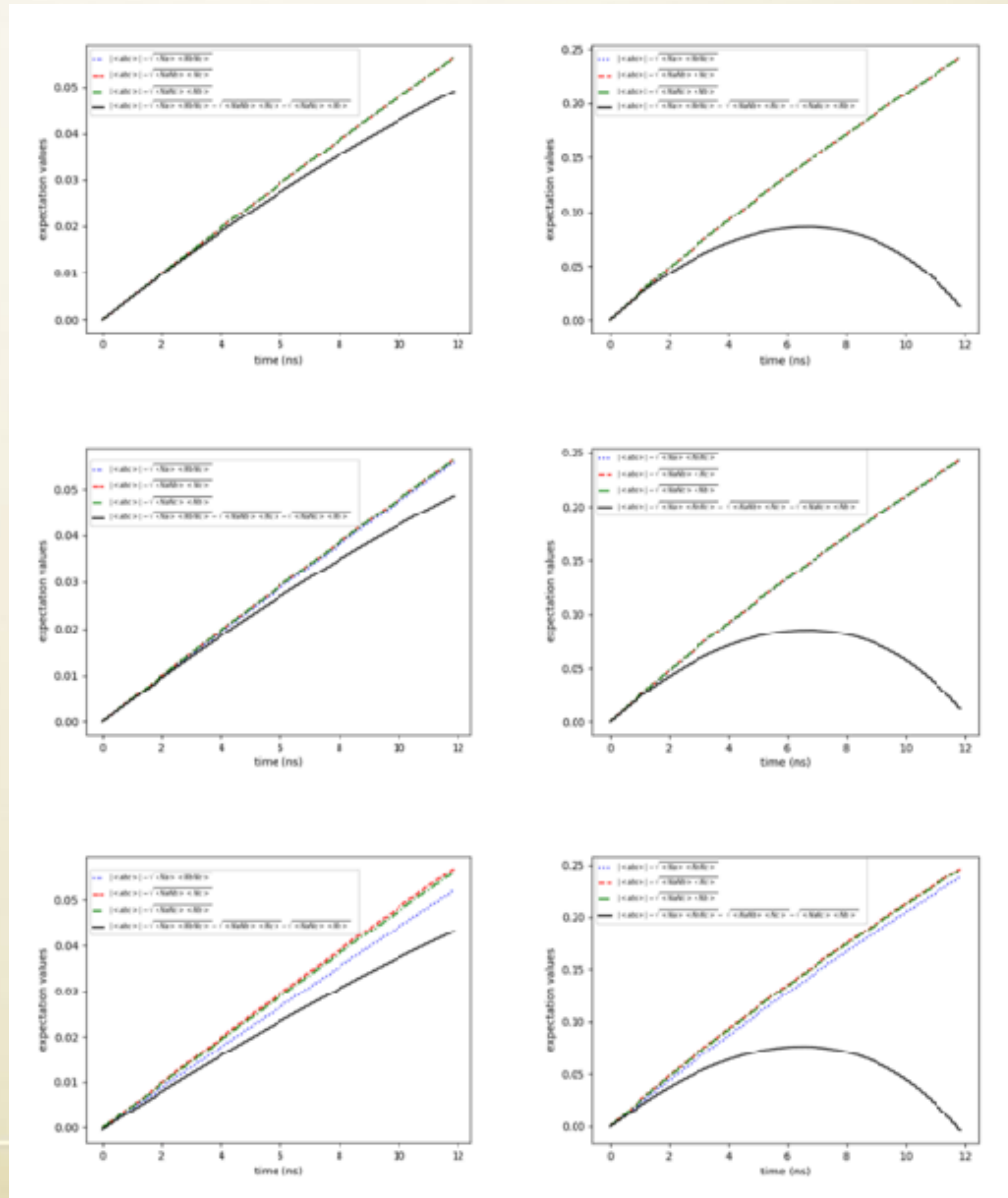

Conclusions

- DCE as a useful resource for QTs: bipartite and multipartite quantum correlations
- Multimode parametric amplification of the quantum vacuum.
- Two different notions of multipartite entanglement emerge: gaussian entanglement with double SPDC or non-gaussian with three-mode SPDC.